CONCLUSION

The proposed method which is based on the combined principle of the resonator and waveguide method can be used with certain modification over a wide range of values of ϵ' and $\tan \delta$. This method can be considered to be universal like waveguide or coaxial line methods [6]–[10].

However, the waveguide and coaxial line methods which can be used to measure ϵ' and $\tan \delta$ over a wide range of values are to employ graphical solution in certain cases in order to solve nonsingle valued transcendental equations [7]. Simpler working equations in closed form in the given case can be considered to be an added advantage of this method over the earlier methods, which enables measurement of ϵ' and $\tan \delta$ over wide range of values.

In certain cases of the earlier methods, fabrication of the experimental sample, in order to satisfy the condition of quasi-stationary distribution of field, requires a very sophisticated technological setup [9]. The same experimental unit cannot always be used for measuring the parameters under two conditions of field distribution [6], [9].

Thus this proposed new method which enables measurement of ϵ' and $\tan \delta$ over a wide range of values can be used with certain attachment for the whole range of these parameter's values. When the size of the sample is conveniently limited the proposed method proves to be superior to the earlier methods. Convenience of thermal shielding of the experimental sample makes it possible to study the dependence of ϵ' and $\tan \delta$ with respect to temperature, which has important scientific significance regarding investigation of the properties of materials.

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A Spectral Domain Analysis for Solving Microstrip Discontinuity Problems

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***Abstract*—A general approach for deriving quasi-static equivalent circuits for discontinuities in microstrip lines is presented. The formulation is based upon Galerkin's method applied in the Fourier transform domain. Numerical results are presented for a number of different configurations and compared with data available from other sources.**

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I. INTRODUCTION

THE PURPOSE of this paper is to develop a general approach for computing the discontinuity capacitance for a wide variety of microstrip structures. Currently, a number of different methods exist for attacking this problem, e.g., the moment method, the variational approach, projective method of solution for the integral equation, to list a few. Discussion of these methods may be found in publications by Farrar and Adams [1], Maeda [2], and Silvester and Benedek [3], [4]. The approach to be described in this paper is somewhat novel in that it uses the familiar Galerkins' method in the Fourier domain rather than the spatial domain. The principal advantage of using the Fourier domain is that the integral equation in the space domain is transformed into an algebraic formulation after Fourier transformation of the convolution form. The method is also quite general and is therefore useful for dealing with a wide variety of discontinuity configurations. We begin with the description of the method in Section II. Illustrative examples and comparison of the results with other experimental and theoretical data will follow.

II. GENERAL FORMULATION OF THE DISCONTINUITY PROBLEM

To illustrate the formulation of the general discontinuity problem, let us consider the geometry shown in Fig. 1. The structure consists of two ground planes (one at the top and the other at the bottom), two dielectric layers with dielectric constants ϵ_1 and ϵ_2, and a set of infinitely thin metallic strips placed at the interface of the two media. The ground planes and dielectric materials are assumed to be lossless and infinite in extent in the x and z directions. Note that no restriction is placed on the shapes of the metallic strips. It is further noted that by letting $h_2 \rightarrow \infty$ and $\epsilon_{r1} = 1$, this structure reduces to the open one that will be considered in the next section.

It will be assumed in this work that the frequency range of interest is such that the quasi-static approximation is valid. In many cases, this approach may be adequate up to perhaps 1 GHz. The potential function ϕ then satisfies Poisson's equation

$$\nabla^2\phi(x,y,z) = -\frac{1}{\epsilon}\rho(x,z)\delta(y-h_1) \qquad (1)$$

where ρ is the charge density distribution on the strips and δ is the familiar Dirac delta. Typically, one transforms this equation into an integral equation via the use of the Green's function. The integral equation is

$$\phi(x,y,z) = \int G(x-x',z-z',y \mid y')\rho(x',z')\,dx'dz' = G^*\rho \qquad (2)$$

where G is the Green's function and * implies convolution. The Green's function appearing in (2) is obtainable by calculating the potential distribution of a point charge located at the interface of the two dielectric layers. The form of the Green's function derived in this manner is, in general, quite complicated due to the inhomogeneous nature of the filling. Also, the Green's function is slowly converging if the separation between the two ground planes becomes large. Finally, if the upper plate is entirely removed and if we are dealing with an unshielded geometry, the waveguide approach for constructing the Green's function is no longer applicable. However, all of these difficulties are circumvented when one works in the Fourier transform domain. This will be evident from the discussion that follows.

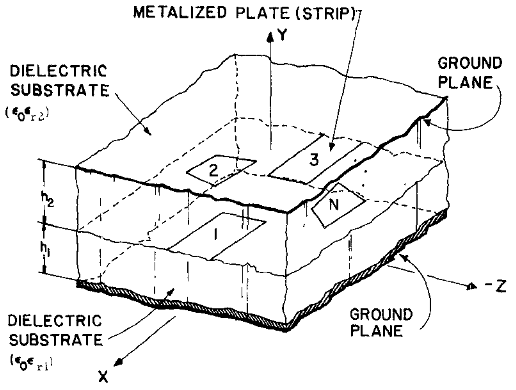

Fig. 1. General microstrip discontinuity problem.

The first step in the method is to introduce the two-dimensional Fourier transform via the following definitions:

$$\tilde{\phi}(\alpha,y,\beta) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \phi(x,y,z)\exp(j\alpha x + j\beta z)\,dx\,dz \qquad (3)$$

$$\phi(x,y,z) = \frac{1}{(2\pi)^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \tilde{\phi}(\alpha,y,\beta)\exp(-j\alpha x - j\beta z)\,d\alpha\,d\beta \qquad (4)$$

of the forward and inverse transforms. The second step is to transform (1) to get

$$-(\alpha^2+\beta^2)\tilde{\phi}(\alpha,y,\beta) + \frac{d^2\tilde{\phi}(\alpha,y,\beta)}{dy^2} = -\frac{1}{\epsilon}\tilde{\rho}(\alpha,\beta)\delta(y-h_1). \qquad (5)$$

The solution to (5), which is a one-dimensional differential equation, is written in a straightforward manner for the two regions above and below the interface:

$$\tilde{\phi}(\alpha,y,\beta) = \begin{cases} A_1(\alpha,\beta)e^{\gamma y} + A_2(\alpha,\beta)\exp(-\gamma y), & 0 \le y \le h_1 \\ B_1(\alpha,\beta)e^{\gamma y} + B_2(\alpha,\beta)\exp(-\gamma y), & h_1 \le y \le h_1 + h_2 \end{cases} \qquad (6)$$

where

$$\gamma = (\alpha^2+\beta^2)^{1/2}.$$

The third step is to apply the boundary condition on $\tilde{\phi}$ in the Fourier transform domain. The boundary conditions in the space domain are: a) ϕ, the potential function equal to 0 on the ground planes; and b) the continuity condition on ϕ at the interface $y = h$. In addition, the normal component of the displacement vector at the interface is required to be discontinuous by the charge distribution at the interface. All of these boundary and interface conditions are readily transformed into the Fourier domain to yield

$$\tilde{\phi}(\alpha,0,\beta) = 0 \tag{7a}$$

$$\tilde{\phi}(\alpha,h_1+h_2,\beta) = 0 \tag{7b}$$

$$\tilde{\phi}(\alpha,h_1^-,\beta) = \tilde{\phi}(\alpha,h_1^+,\beta) \tag{7c}$$

$$\epsilon_{r2}\epsilon_0 \frac{\partial\tilde{\phi}(\alpha,y,\beta)}{\partial y}\bigg|_{h_1^+} - \epsilon_{r1}\epsilon_0 \frac{\partial\tilde{\phi}(\alpha,y,\beta)}{\partial y}\bigg|_{h_1^-} = -\tilde{\rho}(\alpha,h_1,\beta). \tag{7d}$$

Incorporating (7) into (6) allows us to derive the solution

$$\tilde{\phi}(\alpha,h_1,\beta) = \frac{\tilde{\rho}(\alpha,\beta)}{\epsilon_0\gamma[\epsilon_{r2}\coth\gamma h_2 + \epsilon_{r1}\coth\gamma h_1]}. \tag{8}$$

Equation (8) gives the expression for the transform of the potential function in terms of the Fourier transform of the charge distribution at the interface. The factor multiplying $\tilde{\rho}(\alpha,\beta)$ in the right-hand side of (8) may be interpreted as the Fourier transform of the Green's function. It may be noted that the expression for the transform of the Green's function is quite simple and that it goes to a simple limit when one of the ground planes reaches infinity.

We will now discuss the solution of (8) by using the Galerkin's method in the transform domain. To this end we write (8) as

$$\tilde{\phi}_o(\alpha,h_1,\beta) + \tilde{\phi}_i(\alpha,h_1,\beta) = \tilde{G}(\alpha,h_1,\beta)\tilde{\rho}(\alpha,\beta) \tag{9}$$

where

$$\tilde{G}(\alpha,h_1,\beta) = \frac{1}{\epsilon_0\gamma[\epsilon_{r2}\coth\gamma h_2 + \epsilon_{r1}\coth\gamma h_1]} \tag{10}$$

and $\tilde{\phi}_o(\alpha,h_1,\beta)$ is the Fourier transform of the potential distribution in the region of the interface complementary to the strips and $\tilde{\phi}_i(\alpha,h_1,\beta)$ is the corresponding transform of the known potential distribution on the strips. Note that (9) has two unknowns, namely, $\tilde{\rho}$ and $\tilde{\phi}_o$. We are nevertheless able to solve for the desired unknown quantity $\tilde{\rho}$ from the single equation (9). The method of solution is based upon an application of Parseval's theorem [5] and Galerkin's method [5] in connection with (9).

To apply Galerkin's method, it is necessary to introduce a set of basis functions for expanding the unknown $\tilde{\rho}(\alpha,\beta)$. Since the charge distribution $\rho(x,z)$ is nonzero only on the strips, we express $\rho(x,z)$ in terms of $\psi_m^{(n)}(x,z)$, which is 0 outside of the nth strip. The choice of the exact functional form of $\psi_m^{(n)}$ clearly depends upon the nature of the geometry of the problem, and the efficiency of the computation depends upon how well the functions are able to approximate the true charge distribution on the strips. For a complicated configuration, the choice of the basis functions is not trivial and special consideration is needed. For example, for calculating the capacitance of a circular disk, it is found by Itoh and Mittra [6] that the application of the Hankel transform is preferable to the application of the Fourier transform. However, since we will be using a variational approach, the first-order approximation in the charge distribution will give results for the capacitance that would be accurate to the second order.

Having made the choice for the $\psi_m^{(n)}$, we write ρ_n as

$$\rho_n(x,z) = \sum_{m=1}^{M} a_m^{(n)}\psi_m^{(n)}(x,z), \qquad n = 1,2,\cdots,N \tag{11}$$

where $a_m^{(n)}$ are unknown coefficients that must be determined. But since we are considering the problem in the Fourier domain, (11) would be transformed as follows:

$$\tilde{\rho}(\alpha,\beta) = \sum_{n=1}^{N}\sum_{m=1}^{M} a_m^{(n)}\tilde{\psi}_m^{(n)}(\alpha,\beta) \tag{12}$$

where the first summation accounts for the charge distribution on all of the strips. Substituting (12) into (9), one obtains

$$\tilde{\phi}_o(\alpha,h_1,\beta) + \tilde{\phi}_i(\alpha,h_1,\beta) = \tilde{G}(\alpha,\beta)\sum_{n=1}^{N}\sum_{m=1}^{M} a_m^{(n)}\tilde{\psi}_m^{(n)}(\alpha,\beta). \tag{13}$$

Note once again that it is not possible to solve for the unknown coefficients $a_m^{(n)}$ directly from (13) since $\tilde{\phi}_o$ is still unknown. However, the use of the Parseval's theorem would eliminate this unknown as will soon be apparent.

Let us first define the inner product by using the notation

$$\langle\tilde{\psi}_1(\alpha,\beta),\tilde{\psi}_2^*(\alpha,\beta)\rangle = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\tilde{\psi}_1(\alpha,\beta)\tilde{\psi}_2^*(\alpha,\beta)\,d\alpha\,d\beta \tag{14}$$

where the superscript * implies conjugate. We now take the inner product of (13) with $\tilde{\psi}_k^{(l)*}(\alpha,\beta)$ and we write it using the notation of (14). This gives

$$\langle\tilde{\phi}_o(\alpha,\beta) + \tilde{\phi}_i(\alpha,\beta),\tilde{\psi}_k^{(l)*}(\alpha,\beta)\rangle = \sum_{n=1}^{N}\sum_{m=1}^{M} a_m^{(n)}\langle\tilde{G}\tilde{\psi}_m^{(n)},\tilde{\psi}_k^{(l)*}\rangle,$$
$$k = 1,2,\cdots,M$$
$$l = 1,2,\cdots,N. \tag{15}$$

The reader may recognize at this point that we are following the usual Galerkin's procedure except that it is being applied in the Fourier transform domain. Using Parseval's theorem one may write (15) in the following form:

$$\sum_{n=1}^{N}\sum_{m=1}^{M} a_m^{(n)}\langle\tilde{G}\tilde{\psi}_m^{(n)},\tilde{\psi}_k^{(l)*}\rangle = (2\pi)^2\langle\phi_i(x,z),\psi_k^{(l)*}(x,z)\rangle + (2\pi)^2\langle\phi_0(x,z),\psi_k^{(l)*}(x,z)\rangle,$$
$$k = 1,2,\cdots,M$$
$$l = 1,2,\cdots,N. \tag{16}$$

Note that all of the inner products appearing in the above equations are now known except for the second one in the right-hand side. However, it is obvious that this term is 0, because $\psi_l^{(k)}(x,z)$ has zero value on the complementary section of the kth strip on the interface. Finally, using the fact that $\phi_i(x,z)$ is equal to $V^{(i)}$, which is equal to the known potential on the ith strip, we may write the following matrix equation:

$$\sum_{n=1}^{N} \sum_{m=1}^{M} \langle \tilde{G}\tilde{\psi}_m{}^{(n)}, \tilde{\psi}_k{}^{(l)*} \rangle a_m{}^{(n)} = (2\pi)^2 V^{(l)} \int_{S_l} \psi_k{}^{(l)}(x,z)\, dx\, dz, \qquad k = 1,\cdots,M \quad l = 1,\cdots,N. \tag{17}$$

When the $\alpha_m{}^{(n)}$ are determined from the solution of the above matrix equation, one can determine the charge distribution of the nth strip from the equation

$$Q^{(n)} = \sum_{m=1}^{M} a_m{}^{(n)} \int_{S_n} \psi_m{}^{(n)}(x,z)\, dx\, dz, \qquad n = 1,\cdots,N. \tag{18}$$

The capacitance matrix [7] that is defined via the equation

$$\begin{pmatrix} Q^{(1)} \\ \cdot \\ \cdot \\ \vdots \\ Q^{(N)} \end{pmatrix} = \begin{pmatrix} C_{11},\cdots,C_{1N} \\ \cdot \qquad \cdot \\ \cdot \qquad \cdot \\ \cdot \qquad \cdot \\ C_{N1},\cdots,C_{NN} \end{pmatrix} \begin{pmatrix} V^{(1)} \\ \cdot \\ \cdot \\ \cdot \\ V^{(N)} \end{pmatrix} \tag{19}$$

where $Q^{(i)}$ and $V^{(i)}$ are the total charge and potential distribution on the ith strip, can now be computed from the knowledge of the total charge.

Having detailed the general approach to the solution of the capacitance problem, we will now illustrate the application of the theory to some structures of interest.

III. GAP IN THE CENTER CONDUCTOR OF AN UNSHIELDED MICROSTRIP LINE

The geometry of this structure is shown in Fig. 2. It is convenient to represent the equivalent network in terms of a pi circuit, as shown in Fig. 2. The series capacitance C_g may be associated with the coupling phenomena between the two center strips, while the shunt capacitance C_p may be associated with the fringing effect of the edge of the strip. In order to find a relationship between C_p and C_g and C_{11} and C_{12}, we consider the schematic configuration shown in Fig. 3. We note that the structure shown in Fig. 2 corresponds to the case of $L \to \infty$ in Fig. 3(a). C_{oc} represents the excess capacitance (end effect) and C_0 is the capacitance per unit length of the uniform microstrip with the same width. The capacitance matrix of the symmetrical structure shown in Fig. 3(a) may now be written in the following form:

$$\begin{pmatrix} Q^{(1)} \\ Q^{(2)} \end{pmatrix} = \begin{pmatrix} C_{11} & C_{12} \\ C_{12} & C_{11} \end{pmatrix} \begin{pmatrix} V^{(1)} \\ V^{(2)} \end{pmatrix} \tag{20}$$

where $V^{(1)}$ and $V^{(2)}$ are the potentials on the strips, and $Q^{(1)}$ and $Q^{(2)}$ are the induced charges on the corresponding strips. For even excitation, as shown in Fig. 3(b), one

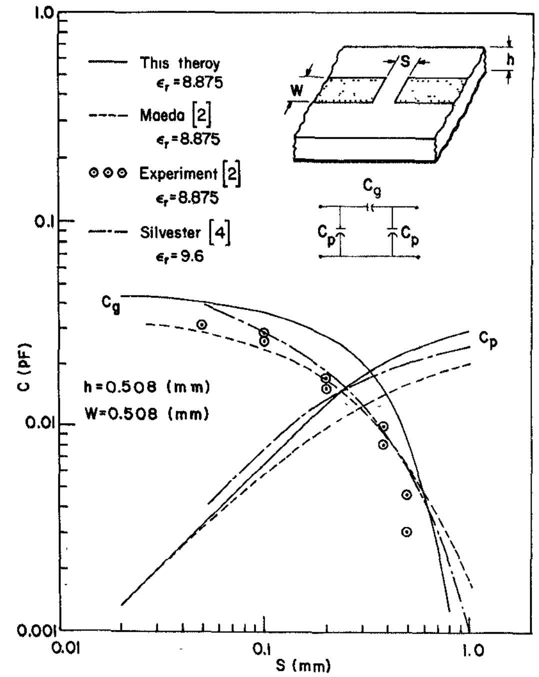

Fig. 2. Series and shunt capacitances for gap discontinuity in microstrip.

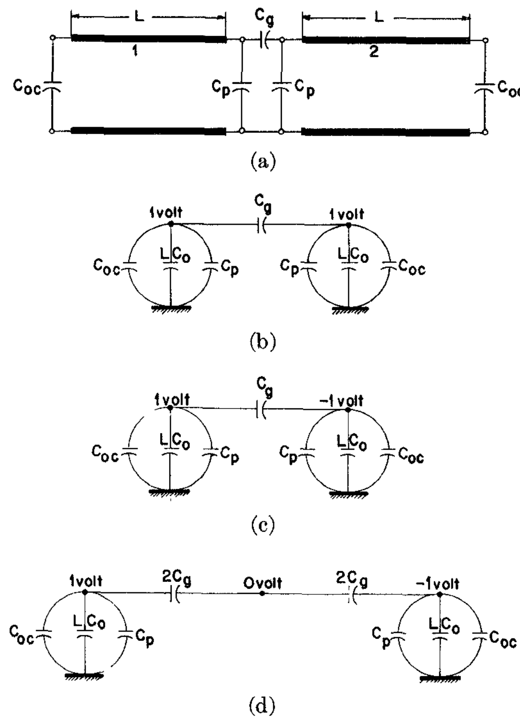

Fig. 3. Lumped circuit representation of a gap structure.

obtains $Q^{(1)} = C_{oc} + LC_0 + C_p$ (note that there is no coupling between strips). From (20) we obtain $Q^{(1)} = C_{11} + C_{12}$; therefore, one may write the following equation:

$$C_{11} + C_{12} = C_{oc} + LC_0 + C_p. \tag{21}$$

Now let us consider the odd excitation as shown in Fig. 3(c) and (d). One readily obtains that $Q^{(1)} = 2C_g + C_p + C_{oc} + LC_0$ (note that the strips are coupled in this case). From (20) we derive that $Q^{(1)} = C_{11} - C_{12}$. Therefore

$$C_{11} - C_{12} = 2C_g + C_p + C_{oc} + LC_0. \quad (22)$$

Recall that we have concentrated on evaluating the equivalent circuit for the structure with finite L. To obtain the equivalent circuit for the infinite structure, one considers the limiting procedure of L becoming infinitely large. By subtracting and adding (21) and (22) one obtains

$$C_g = -\lim_{L\to\infty} C_{12} \quad (23)$$

and

$$C_p = \lim_{L\to\infty} (C_{11} + C_{12} - C_{oc} - LC_0). \quad (24)$$

It is clear from the above relations that C_{11} and C_{12} can be determined once we solve for the total charge on the two strips. To solve the latter problem we now refer to the general method outlined in Section II and apply it to the specific problem at hand. To this end we first derive the transformed Green's function by letting $\epsilon_{r2} = 1$, $\epsilon_{r1} = \epsilon_r$, $h_1 = h$, and $h_2 \to \infty$ in (10), which reads

$$\tilde{G}(\gamma,h) = \frac{1}{\epsilon_0\gamma[\epsilon_r \coth \gamma h + 1]}. \quad (25)$$

This equation is identical to the Green's function derived in the earlier work [8]. The next step is to choose the basis functions $\psi^{(i)}$. For the problem under consideration a simple but adequate choice is a two-dimensional pulse function. When we use these functions, we have

$$\psi_1^{(1)}(x,z) = \left[H\left(x + \frac{W}{2}\right) - H\left(x - \frac{W}{2}\right)\right] \cdot \left[H\left(z - \frac{S}{2}\right) - H\left(z - L - \frac{S}{2}\right)\right] \quad (26a)$$

$$\psi_1^{(2)}(x,z) = \left[H\left(x + \frac{W}{2}\right) - H\left(x - \frac{W}{2}\right)\right] \cdot \left[H\left(z + L + \frac{S}{2}\right) - H\left(z + \frac{S}{2}\right)\right] \quad (26b)$$

where $H(x)$ is the conventional step function, i.e.,

$$H(x) = \begin{cases} 1, & x > 0 \\ 0, & x < 0. \end{cases}$$

Fourier transforms of the above basis functions are readily obtained and are written as

$$\tilde{\psi}_1^{(1)}(\alpha,\beta) = \frac{4}{\alpha\beta} \sin\frac{\alpha W}{2} \sin\frac{\beta L}{2} \exp\left[j\beta\left(\frac{S}{2} + \frac{1}{2}\right)\right] \quad (27a)$$

$$\tilde{\psi}_1^{(2)}(\alpha,\beta) = \tilde{\psi}_1^{(1)*}(\alpha,\beta). \quad (27b)$$

We now have the necessary information for evaluating the matrix elements in (17), and since the matrix size is only 2×2, the solution for the two coefficients $a_1^{(1)}$ and $a_1^{(2)}$ is a trivial step. Knowledge of $a_1^{(1)}$ and $a_1^{(2)}$ allows us to calculate the total charge on each strip and obtain C_{11} and C_{22} from (20). The other two capacitances, namely, C_p and C_g, may be computed from (23) and (24). Although we are interested in the computed results for the case $L \to \infty$, it was only necessary to compute the numerical result for $L \geq 10h$. It should also be pointed out that the computation of C_{11} and C_{12} has to be accurate in order to get precise results for C_p and C_g. This is due to the fact that the computation of latter quantities requires the subtraction of two large numbers that are nearly equal. It is reported by Silvester and Benedek [3] that the subtraction of two nearly equal numbers is avoided in their method.

Fig. 2 shows the graphical plot for C_p and C_g for $\epsilon_r = 8.875$. Those results, which are obtained by the method just described, are also compared with those of Maeda [2], who used the dielectric Green's function, and the agreement is seen to be fairly good. As expected, C_p approaches a constant in the limit of large separation distance S, the limiting value being equal to the open-circuit capacitance of the semi-infinite transmission line. These results are also compared with those of Benedek and Silvester [4], who obtained the gap capacitance for $\epsilon_r = 9.6$.

IV. CAPACITANCE OF A FINITE SECTION OF MICROSTRIP LINE

We now consider the structure shown in Fig. 4, which is a finite section of a microstrip line of length L, width W, and height h above the ground plane. Once again the method outlined in Section II can be used for computing the capacitance of the structure that is obtained by letting

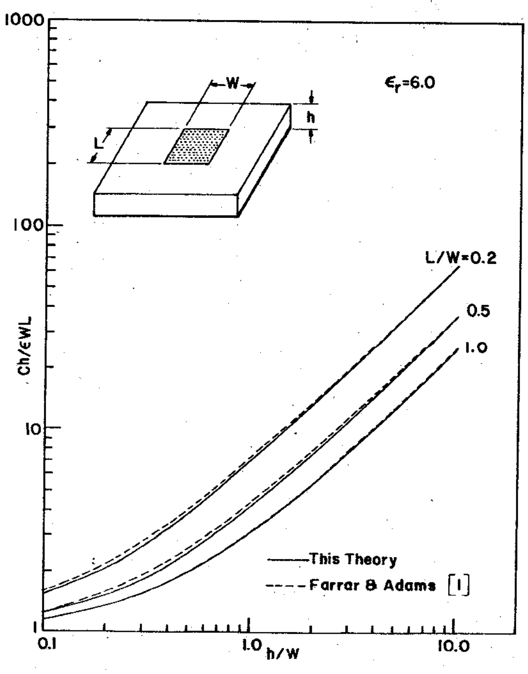

Fig. 4. Normalized capacitance of finite section of microstrip line.

$S = 0$ in the geometry shown in Fig. 2. Thus the computation may be performed by letting $S \rightarrow 0$, replacing L by $L/2$ and setting $V^{(1)} = V^{(2)} = 1$ V in the formulas used in the previous section. It may also be noted that the computation of the capacitance is much simpler in the present case because it does not involve the subtraction of nearly equal large numbers. For the same reason, the accuracy of the results is also better in the present case as compared to the gap discontinuity results.

Fig. 4 shows the normalized capacitance $Ch/\epsilon_0\epsilon_r LW$ of a rectangular microstrip section versus W/h for a range of L/W between 0.2 and 1.0. Note that $\epsilon_0\epsilon_r LW/h$ is the classical parallel-plate capacitance (neglecting the fringing effect). As expected, the normalized capacitance asymptotically approaches 1 as h/W becomes small. Although only one-term expansion has been used to derive the result, it is evident that the computed values compare very well with those obtained by Farrar and Adams [1] via the method of moments.

V. THE END EFFECT IN AN OPEN-ENDED MICROSTRIP LINE

Fig. 5 shows the geometry to be considered, namely, a semi-infinite open-ended microstrip line. The effect of the discontinuity at the open end may be represented by the lumped shunt capacitance C_{oc} [3]. The procedure for computing C_{oc} is as follows. One computes the total capacitance $C_t(L)$ of a rectangular-section microstrip line of length L. The excess capacitance is then obtained using the formula

$$C_{oc} = \lim_{L\rightarrow\infty} C_{\text{exc}}(L) = \lim_{L\rightarrow\infty} \tfrac{1}{2}[C_t(L) - LC_0] \qquad (28)$$

where C_0 is again the capacitance per unit length of an infinitely long microstrip line with same h and W parameters. The justification for the use of (28) is easily understood by referring to the lumped circuit representation of the open-circuited microstrip line as shown in Fig. 6. It is again necessary to subtract two large nearly equal numbers, viz., $C_t(L)$ and LC_0 when using (28). This difficulty may be decreased somewhat by using numerical values of C_0 in (28) that were computed using a three-dimensional formulation. In the computation, the following expressions were used:

$$C_{oc}(L + \delta) = \tfrac{1}{2}[C_t(t + \delta) - (L + \delta)C_0]. \qquad (29)$$

For large L, (29) gives the value of C_0:

$$C_0 = \frac{C_t(L + \delta) - C_t(L)}{\delta} \qquad (30)$$

where we have used the fact $C_{oc}(L + \delta) \simeq C_{oc}(L)$. The results for C_0 were compared with those obtained by Yamashita and Mittra [9] who used the variational technique for two-dimensional structure; good agreement between the two results was observed. Using these values of C_0 from (30) and computing C_{oc} from (28), one obtains the results that are shown in Fig. 5 for range of W/h and

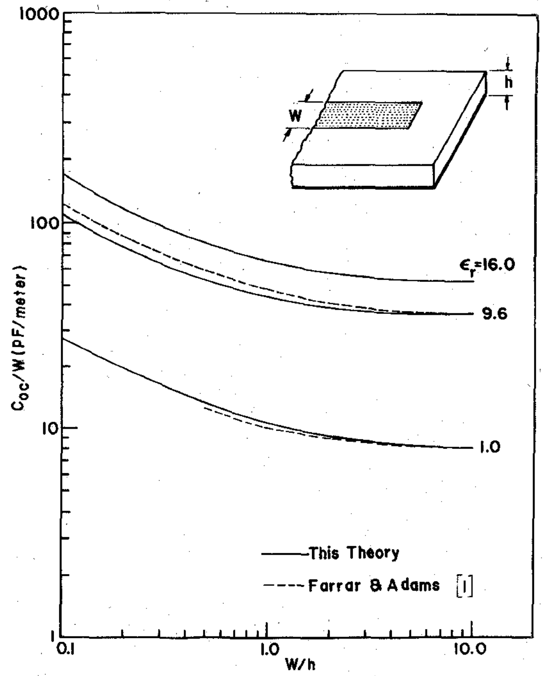

Fig. 5. Excess capacitance of open-circuited microstrip line.

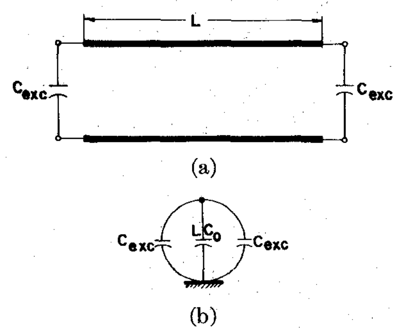

Fig. 6. Lumped circuit representation of open-circuited microstrip.

number of different dielectric constants. Once again the results have been compared to those given by Farrar and Adams [1], and although only one-term approximation was used in our computation, the results nevertheless show very good agreement with those obtained by the moment method. The results have also been compared with those obtained by Silvester and Benedek [3], though not shown in Fig. 5. It is found that their results are 4–8 percent larger than those obtained by the present method. The calculations were performed on a CDC G-20 (this computer is about ten times slower than the IBM 36-/75). It is found that about 150 s are required for total processing time for evaluating each open capacitance.

VI. CONCLUSIONS

In this paper we have described a general approach for computing the quasi-static equivalent circuit for microstrip line discontinuities. The approach is novel in that Galerkin's method is employed in the spectral domain to derive a matrix equation that is numerically efficient for solving the capacitance problem. Numerical results have been obtained and compared with previously published results. The accuracy and the relative efficiency of the method have been demonstrated.

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Theory of Coupled Open Transmission Lines and Its Applications

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***Abstract*—A technique is presented which is applicable to any uniform coupled open transmission lines such as coupled optical integrated circuits. The proposed technique is as follows.**

The electromagnetic fields of the wave propagating along a coupled line is expressed in terms of a linear combination of the fields associated with the individual lines, as a zero-order approximation. Inserting this trial field description into the variational expression for the propagation constant β, and applying the well-known Rayleigh–Ritz's procedure, accurate solutions for the propagation constants of the coupled lines are obtained.

This method can be applied generally to analyze coupled structures in microwave, millimeter wave, and optical wave circuitry. As an illustrative example, the coupling between two optical transmission lines consisting of lens-like dielectric media has been analyzed by means of the proposed technique.

I. INTRODUCTION

THE PROBLEM of coupling between open transmission lines is interesting both from academic and practical points of view in connection with the design and analysis of optical integrated circuits and components (see Fig. 6).

Though several papers concerning the coupling of open transmission lines have been reported [1]–[4], only the special case where the coupling occurs between two identical open transmission lines has been analyzed. To the authors knowledge, a technique adequate to treat the coupling between two different open transmission lines has not been given before.

This paper presents a theory which can be applied to two arbitrary coupled open transmission lines. The technique proposed is based on the variational method.[1] The procedure of the calculation is quite simple and straightforward as long as the electromagnetic fields associated with individual transmission lines are already known. As an example of the application of the proposed theory, the coupling between two dielectric lines consisting of lens-like media has been analyzed.

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II. VARIATIONAL EXPRESSION FOR THE PROPAGATION CONSTANT

The magnetic field $\boldsymbol{H}$ of any uniform open transmission line can be expressed as

$$\boldsymbol{H} = (\boldsymbol{h}_t + \boldsymbol{i}_z h_z) \exp[j(\omega t - \beta z)] \tag{1}$$

where $\boldsymbol{h}_t$ and h_z are the transverse and longitudinal components of the field, respectively, $\boldsymbol{i}_z$ is a unit vector in the longitudinal direction z, and β is the propagation constant. The variational expression for the propagation constant β in the z direction of a lossless uniform transmission line is given as[2]

$$\beta^2 = N/D \tag{2}$$

[1] The theory described in the present paper was originally reported at Radiation Science Research Committee on April 30, 1971, in Japanese. After preparing the manuscript of the present paper, the authors found two related articles. One is Marcuse's work [5] where the coupling problem is treated with a perturbation method, and the other is Snyder's paper [6] in which the problem is solved by a modal-expansion approach.

[2] The variational expressions for the propagation constant of the guided waves have been derived by several authors in different forms. The expression (2) is a slight modification and generalization of Kurokawa's original one [7]. The variational expression (2) has the advantage that it can be easily applied even though the material constants involved change discontinuously in the transverse cross-sectional surface of the guide.